Worksheet 4

1. Prove $\frac{d}{dx}(x^n) = n * x^{n-1}$ (power rule in calculus) for each natural number.

   Recall: Product rule in calculus $\frac{d}{dx}(u * v) = u' * v + u * v'$ (You may want to use this fact in the inductive step)

2. Prove that $n! > 2^n$ for all $n \geq 4$

3. Define a sequence $a_1 = 1, a_2 = 3, a_n = 2a_{n-1} - a_{n-2}$ when $n \geq 3$

   Prove for $a_n = 2n - 1$ for each natural number n.

4. Prove that for each natural number n, $1 + 3 + 5 + \cdots + (2n - 1) = n^2$